A New Saturated Nonlinear PID Global Regulator for Robot Manipulators: Passivity Based Analysis⋆

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Abstract— In this paper we propose a saturated nonlinear PID regulator for solving the problem of global regulation in robot manipulators with bounded torques. An approach based on interconnected passive systems is used for analyzing the global asymptotic stability. To this end, we use a passivity theorem which is an adaptation of a passivity theorem given in Khalil [21]. Such a theorem deals with asymptotic stability of the equilibrium of an unforced interconnected system in which the feedforward system is state strictly passive and the feedback system is passive and equilibrium–state observable.

I. INTRODUCTION

Saturation is one of the most commonly encountered nonlinearities in robot control systems. This phenomenon is present when the actuators are driven by sufficiently large control signals. If this physical constraint is not considered in the controller design, it may lead to a lack of stability guarantee. Some works have been reported to solve this problem, [1]–[8]. On the other hand, some global nonlinear PID regulators, which are based on Lyapunov and passivity theory, has been reported in [9]–[12], however, they do not take into account the effects of actuators saturation. Recently, two saturated PID controllers have been reported: a semiglobal saturated linear PID control [13] and a global saturated nonlinear PID control [14].

In this paper we introduce a new global saturated nonlinear PID controller, which has a simpler structure than that presented in [14]. It is demonstrated that the proposed saturated nonlinear PID regulator, can be considered as a passivity based regulator that allows to see the closed loop system as a feedback connection between two passive systems. With the end of proving the asymptotic stability of the proposed controller, we present a passivity theorem which is an adaptation, for asymptotic stability purposes, of a passivity theorem given in Khalil [21]. Such a theorem deals with the asymptotic stability of the equilibrium of an unforced interconnected system in which the feedforward system is state strictly passive, and the feedback system is passive and observable in the equilibrium–state.

Throughout this paper, we use the notation $\lambda_{\rm m}\{A\}$ and $\lambda_{\rm M}\{A\}$ to indicate the smallest and largest eigenvalues, respectively, of a symmetric positive definite bounded matrix $A(\boldsymbol{x})$, for any $\boldsymbol{x} \in {\rm I\!R}^n$. The norm of vector $\boldsymbol{x}$ is defined as $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$ and that of matrix A is defined as the corresponding induced norm $\|A\| = \sqrt{\lambda_{\rm M}\{A^TA\}}$. L_2^n and L_{2e}^n denote the space of n–dimensional square integrable functions and its extension, respectively.

II. DYNAMICS OF RIGID ROBOTS AND CONTROL PROBLEM FORMULATION

The dynamics of a serial n-link rigid robot, including the effect of viscous friction, can be written as [15]:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + F_v\dot{\boldsymbol{q}} = \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{q}$ is the $n \times 1$ vector of joint displacements, $\dot{\boldsymbol{q}}$ is the $n \times 1$ vector of joint velocities, $\boldsymbol{\tau}$ is the $n \times 1$ vector of applied torques, $M(\boldsymbol{q})$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ is the $n \times n$ matrix of centripetal and Coriolis torques, F_v is the $n \times n$ diagonal matrix of viscous friction coefficients f_{vi} for $i = 1, 2, \ldots, n$, and $\boldsymbol{g}(\boldsymbol{q})$ is the $n \times 1$ vector of gravitational torques obtained as the gradient of the robot potential energy $\mathcal{U}(\boldsymbol{q})$, i.e.

$$\boldsymbol{g}(\boldsymbol{q}) = \frac{\partial \mathcal{U}(\boldsymbol{q})}{\partial \boldsymbol{q}}. \tag{2}$$

We assume that the links are jointed together with revolute joints.

A. Properties of the Robot Dynamics

Three important properties of dynamics (1) are the following:

Property 1. [16] The matrix $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ and the time derivative $\dot{M}(\boldsymbol{q})$ of the inertia matrix satisfy:

$$\dot{\boldsymbol{q}}^T\left[\frac{1}{2}\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q},\dot{\boldsymbol{q}})\right]\dot{\boldsymbol{q}} = \boldsymbol{0} \qquad \forall\ \boldsymbol{q},\dot{\boldsymbol{q}} \in {\rm I\!R}^n.$$

◇

Property 2. [17]. The gravitational torque vector $\boldsymbol{g}(\boldsymbol{q})$ is bounded for all $\boldsymbol{q} \in {\rm I\!R}^n$. This means that there exist finite

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constants $\bar{g}_i \geq 0$ such that

$$\sup_{\boldsymbol{q}\in\mathbb{R}^n} |g_i(\boldsymbol{q})| \leq \bar{g}_i \qquad i = 1, \cdots, n. \tag{3}$$

where $g_i(\boldsymbol{q})$ stands for the elements of $\boldsymbol{g}(\boldsymbol{q})$. Equivalently, there exists a constant k' such that

$$\|\boldsymbol{g}(\boldsymbol{q})\| \ \leq \ k' \quad \text{for all} \quad \boldsymbol{q} \in \mathbb{R}^n.$$

Furthermore there exists a positive constant k_g such that

$$\|\boldsymbol{g}(\boldsymbol{x}) - \boldsymbol{g}(\boldsymbol{y})\| \leq k_g \|\boldsymbol{x} - \boldsymbol{y}\|.$$

for all $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$. $\diamond$

Property 3. In relation with the dynamics model (1). The operator

$$\begin{aligned} H_R \quad &: \quad L_{2e}^n \to L_{2e}^n \\ &: \quad \boldsymbol{\tau} \mapsto \dot{\boldsymbol{q}} \end{aligned}$$

is output strictly passive [18],[19].

B. Problem Formulation

Consider the robot dynamic model (1). Assume that each joint actuator is able to supply a known maximum torque $\tau_i^{\max}$ so that:

$$|\tau_i| \leq \tau_i^{\max}, \qquad i = 1, \cdots, n \tag{4}$$

where τ_i stands for the i–entry of vector $\boldsymbol{\tau}$. We also assume that the maximum torque $\tau_i^{\max}$ of each actuator satisfies the following condition

$$\tau_i^{\max} > \bar{g}_i, \tag{5}$$

where $\bar{g}_i$ was defined in Property 2. This assumption means that the robot actuators are able to supply torques in order to hold the robot at rest for all desired joint position $\boldsymbol{q}_d \in \mathbb{R}^n$.

The control problem is to design a controller to compute the torque $\boldsymbol{\tau} \in \mathbb{R}^n$ applied to the joints, which satisfies the constraints (4), such that, the robot joint displacements $\boldsymbol{q}$ tend asymptotically toward the constant desired joint displacements $\boldsymbol{q}_d$.

III. PASSIVITY DEFINITIONS

Consider dynamical systems represented by

$$\begin{aligned} \dot{\boldsymbol{x}} \quad &= \quad \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) &(6)\\ \boldsymbol{y} \quad &= \quad \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{u}) &(7)\end{aligned}$$

where $\boldsymbol{u} \in \mathbb{R}^n$, $\boldsymbol{y} \in \mathbb{R}^n$, $\boldsymbol{x} \in \mathbb{R}^m$, $\boldsymbol{f}(\boldsymbol{x}^*, \boldsymbol{0}) = \boldsymbol{0}$, $\boldsymbol{h}(\boldsymbol{x}^*, \boldsymbol{0}) = \boldsymbol{0}$, and $\boldsymbol{x}^*$ is the equilibrium point of (6). Moreover $\boldsymbol{f}$, $\boldsymbol{h}$ are supposed sufficiently smooth such that the system is well–defined, i.e., $\forall\ \boldsymbol{u} \in L_{2e}^n$ and $\boldsymbol{x}(\boldsymbol{0}) \in \mathbb{R}^m$ we have that the solution $\boldsymbol{x}(\cdot)$ is unique and $\boldsymbol{y} \in L_{2e}^n$. The following definitions 1 and 2, have been adequate (for non–zero equilibrium) from [21]. $\square$

Definition 1. The system (6)–(7) is said to be passive if there exists a continuously differentiable positive semidefinite function $V(\boldsymbol{x}-\boldsymbol{x}^*)$ (called the storage function) such that

$$\boldsymbol{u}^T\boldsymbol{y} \geq \dot{V}(\boldsymbol{x} - \boldsymbol{x}^*) + \epsilon\|\boldsymbol{u}\|^2 + \delta\|\boldsymbol{y}\|^2 + \rho\psi(\boldsymbol{x} - \boldsymbol{x}^*) \tag{8}$$

where ϵ, δ, and ρ are nonnegative constants, and $\psi(\boldsymbol{x} - \boldsymbol{x}^*) : \mathbb{R}^m \to \mathbb{R}$ is a positive definite function of $\boldsymbol{x} - \boldsymbol{x}^*$. The term $\rho\psi(\boldsymbol{x} - \boldsymbol{x}^*)$ is called the state dissipation rate. Furthermore, the system is said to be: lossless if (8) is satisfied with equality and $\epsilon = \delta = \rho = 0$; that is, $\boldsymbol{u}^T\boldsymbol{y} = \dot{V}(\boldsymbol{x} - \boldsymbol{x}^*)$; input strictly passive if $\epsilon > 0$ and $\delta = \rho = 0$; output strictly passive if $\delta > 0$ and $\epsilon = \rho = 0$; state strictly passive if $\rho > 0$ and $\epsilon = \delta = 0$. If more than one of the constants ϵ, δ, ρ are positive we combine names. $\square$

Definition 2. The system (6)–(7) is said to be equilibrium–state observable if $\boldsymbol{u}(t) \equiv \boldsymbol{0}$ and $\boldsymbol{y}(t) \equiv \boldsymbol{0} \Rightarrow \boldsymbol{x}(t) \equiv \boldsymbol{x}^*$. Equivalently, no solutions of $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{0})$ can stay identically in $S = \{\boldsymbol{x} \in \mathbb{R}^m : \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{0}) = \boldsymbol{0}\}$, other than the solution $\boldsymbol{x}(t) \equiv \boldsymbol{x}^*$. $\square$

Definition 3. $\mathcal{F}(m, \varepsilon, \boldsymbol{x})$ with $1 \geq m > 0$, $\varepsilon > 0$ and $\boldsymbol{x} \in \mathbb{R}^n$ denotes the set of all continuous differentiable increasing functions $\boldsymbol{f}(\boldsymbol{x}) = [\, f(x_1) \quad f(x_2) \quad \cdots \quad f(x_n)\,]^T$ such that

- $|x| \geq |f(x)| \geq m|x|, \quad \forall x \in \mathbb{R} : |x| < \varepsilon$
- $\varepsilon \geq |f(x)| \geq m\varepsilon, \quad \forall x \in \mathbb{R} : |x| \geq \varepsilon$
- $1 \geq \frac{df(x)}{dx} \geq 0, \quad \forall x \in \mathbb{R}$

$\square$

Definition 4. The hard saturation function $\mathbf{SAT}(\boldsymbol{x}; \boldsymbol{k}) \in \mathbb{R}^{\mathbf{n}}$ is defined by

$$\mathbf{SAT}(\boldsymbol{x}; \boldsymbol{k}) = \begin{bmatrix} \mathrm{SAT}(x_1; k_1) \\ \mathrm{SAT}(x_2; k_2) \\ \vdots \\ \mathrm{SAT}(x_n; k_n) \end{bmatrix}, \ \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \ \boldsymbol{k} = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix}$$

where k_i is the i–th saturation limit, and

$$\mathrm{SAT}(x_i; k_i) = \begin{cases} x_i & \text{if} \quad |x_i| \leq k_i \\ k_i & \text{if} \quad x_i > k_i \\ -k_i & \text{if} \quad x_i < -k_i \end{cases} \qquad \text{for} \quad i = 1, 2, \ldots n.$$

Property 4. The integral of a hard saturation function $\int_b^x [\mathrm{SAT}(\xi; k) - b]\, d\xi$ is a positive definite function with an unique and global minimum at $x = b$, with $|b| < k$.

IV. SATURATED NONLINEAR PID GLOBAL REGULATOR

In this section we present a new saturated nonlinear PID controller to solve the set-point control problem of robot manipulators with actuator torque constraints.

A. Main Result

The proposed control law is given by

$$\boldsymbol{\tau} = K_p\mathbf{SAT}\ (\tilde{\boldsymbol{q}}; \boldsymbol{\tau}_p) - K_v\mathbf{SAT}\ (\dot{\boldsymbol{q}}; \boldsymbol{\tau}_v) + K_i\mathbf{SAT}(\boldsymbol{w}; \boldsymbol{\tau}_w) \tag{9}$$

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with

$$\boldsymbol{w} = \int_0^t (\alpha \mathbf{sat}(\tilde{\boldsymbol{q}}(\sigma)) - \dot{\boldsymbol{q}})\, d\sigma$$

where $\boldsymbol{\tau}_p$, $\boldsymbol{\tau}_v$ and $\boldsymbol{\tau}_w$ are the respective vectors of saturation limits satisfying

$$\begin{aligned} k_{p_i}\tau_{p_i} &\le \tau_{p_i}^{\max} \\ k_{v_i}\tau_{v_i} &\le \tau_{v_i}^{\max} \\ k_{i_i}\tau_{w_i} &\le \tau_{w_i}^{\max} \end{aligned}$$

and $\tau_i^{\max} \ge \tau_{p_i}^{\max} + \tau_{v_i}^{\max} + \tau_{w_i}^{\max} \ge \overline{g}_i$, for $i = 1, 2, \ldots n$. and K_p, K_v and K_i are $n \times n$ diagonal positive definite matrices whose element are k_{p_i}, k_{v_i}, k_{i_i} respectively with $i = 1, 2...n$, $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$ denotes the position error vector, $\mathbf{sat}(\tilde{\boldsymbol{q}})$ was defined in Definition 3, α is a small positive constant suitably selected. $\mathbf{SAT}(\tilde{\boldsymbol{q}}; \boldsymbol{\tau}_p) \in \mathbb{R}^n$, $\mathbf{SAT}(\dot{\boldsymbol{q}}; \boldsymbol{\tau}_v) \in \mathbb{R}^n$ and $\mathbf{SAT}(\boldsymbol{w}; \boldsymbol{\tau}_w) \in \mathbb{R}^n$ are the proportional, derivative and integral hard saturation functions respectively defined in Definition 4. The closed loop system is shown in the Figure 1.

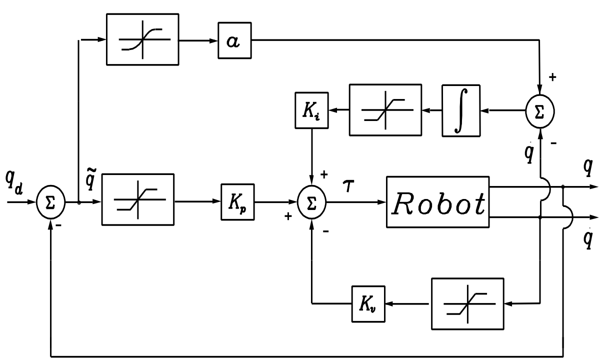

Fig. 1. Closed loop system

For analysis purpose, the control law (9), can be written as $\boldsymbol{\tau} = K_p\mathbf{SAT}(\tilde{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q_d}) + \boldsymbol{\tau}'$ where $\boldsymbol{\tau}' = -K_v\mathbf{SAT}(\dot{\boldsymbol{q}}) + \boldsymbol{\tau}''$ and $\boldsymbol{\tau}'' = K_i\mathbf{SAT}(\boldsymbol{w}) - \boldsymbol{g}(\boldsymbol{q}_d)$ (for cumbersome notation reasons, henceforth, we omit the saturation limit parameters). This structure of the control law, allow us, to represent the closed loop system as an unforced interconnected system, (see Figure 2).

In the next paragraphs we analyze the stability of the equilibrium of the closed loop system formed by (9) and (1), which is given by

$$\frac{d}{dt}\begin{bmatrix} \tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}} \\ \boldsymbol{w} \end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M^{-1}\left[K_p\mathbf{SAT}(\tilde{\boldsymbol{q}}) - K_v\mathbf{SAT}(\dot{\boldsymbol{q}}) \right. \\ \left. + K_i\mathbf{SAT}(\boldsymbol{w}) - F_v\dot{\boldsymbol{q}} - C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q})\right] \\ \alpha\mathbf{sat}(\tilde{\boldsymbol{q}}) - \dot{\boldsymbol{q}} \end{bmatrix} \tag{10}$$

which is an autonomous differential equation whose unique equilibrium is: $\begin{bmatrix}\tilde{\boldsymbol{q}}^T & \dot{\boldsymbol{q}}^T & \boldsymbol{w}^T\end{bmatrix}^T = \begin{bmatrix}0 & 0 & K_i^{-1}\boldsymbol{g}(\boldsymbol{q}_d)\end{bmatrix}^T$, provided that $\lambda_{\mathrm{m}}\{K_i\} > \max_i \bar{g}_i$. Such an analysis is carried out using passivity theory of interconnected systems.

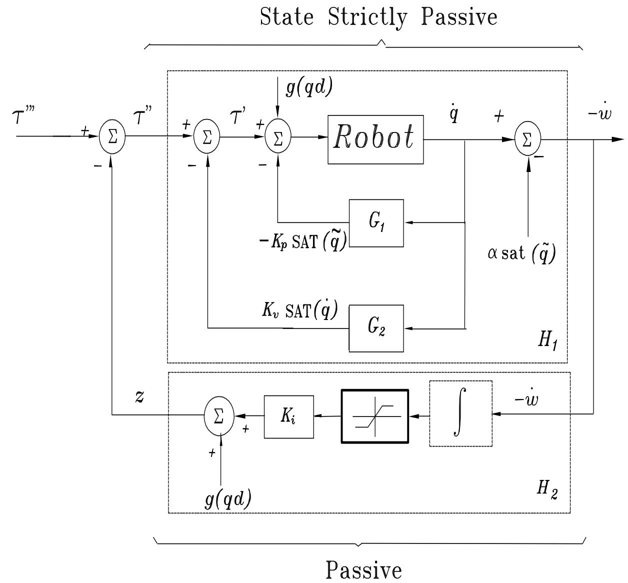

Fig. 2. Feedback System

Now, we are in position to introduce some propositions that will allow us to conclude asymptotic stability of the closed loop system (10).

Proposition 1. Robot dynamics (1) in closed-loop with

$$\boldsymbol{\tau} = K_p\mathbf{SAT}(\tilde{\boldsymbol{q}}) - K_v\mathbf{SAT}(\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}_d) + \boldsymbol{\tau}'' \tag{11}$$

is state strictly passive (see Figure 2), from the input torque $\boldsymbol{\tau}''$ to the output $-\dot{\boldsymbol{w}} = (\dot{\boldsymbol{q}} - \alpha\,\mathbf{sat}(\tilde{\boldsymbol{q}}))$, it is to say,

$$(\dot{\boldsymbol{q}} - \alpha\,\mathbf{sat}(\tilde{\boldsymbol{q}}))^T\boldsymbol{\tau}'' \ge \dot{V}_{1b}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) + \varphi(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}), \tag{12}$$

with the storage function given by

$$\begin{aligned} V_{1b}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) &= \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} - \alpha\,\mathbf{sat}(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\dot{\boldsymbol{q}} + \mathcal{U}(\boldsymbol{q}) \\ &\quad -\mathcal{U}(\boldsymbol{q}_d) + \sum_{i=1}^{n} k_{p_i}\int_0^{\tilde{q}_i} \mathrm{SAT}(\xi_i)\, d\xi_i \\ &\quad +\boldsymbol{g}(\boldsymbol{q}_d)^T\tilde{\boldsymbol{q}} + \sum_{i=1}^{n}\alpha\int_0^{\tilde{q}_i} f_{v_i}\mathrm{sat}(\xi_i)\, d\xi_i, \end{aligned} \tag{13}$$

where the term $\sum_{i=1}^{n}\alpha\int_0^{\tilde{q}} f_{vi}\mathrm{sat}(\xi_i)\, d\xi_i$ is related with the dissipated energy by the viscous friction torque, and $\alpha\,\mathbf{sat}(\tilde{\boldsymbol{q}})M(\boldsymbol{q})\dot{\boldsymbol{q}}$ is a cross term which depends on position error and velocity. The state dissipation rate is given by:

$$-\varphi(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) =$$

$$\begin{aligned} &-\dot{\boldsymbol{q}}^T K_v\mathbf{SAT}(\dot{\boldsymbol{q}}) - \dot{\boldsymbol{q}}^T F_v(\dot{\boldsymbol{q}}) - \alpha\dot{\mathbf{sat}}(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &-\alpha\mathbf{sat}(\tilde{\boldsymbol{q}})^T C(\boldsymbol{q},\dot{\boldsymbol{q}})^T\dot{\boldsymbol{q}} + \alpha\mathbf{sat}(\tilde{\boldsymbol{q}})^T K_v\mathbf{SAT}(\dot{\boldsymbol{q}}) \\ &-\alpha\mathbf{sat}(\tilde{\boldsymbol{q}}^T)[\boldsymbol{g}(\boldsymbol{q}_d) - \boldsymbol{g}(\boldsymbol{q})] - \alpha\mathbf{sat}(\tilde{\boldsymbol{q}}^T)K_p\mathbf{SAT}(\tilde{\boldsymbol{q}}). \end{aligned}$$

Proof. The system (1) in closed-loop with the control law (11) is given by (see Figure 2)

$$\frac{d}{dt}\begin{bmatrix}\tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}}\end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M^{-1}\left[K_p\mathbf{SAT}(\tilde{\boldsymbol{q}}) - K_v\mathbf{SAT}(\dot{\boldsymbol{q}}) + \boldsymbol{\nu}\right]\end{bmatrix} \tag{14}$$

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$$+\begin{bmatrix} \mathbf{0} \\ M^{-1}\boldsymbol{\tau}'' \end{bmatrix}$$

where $\boldsymbol{\nu} = \boldsymbol{g}(\boldsymbol{q}_d) - F_v\dot{\boldsymbol{q}} - C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q})$. By using the Property 4 and the procedure given in [22], it is possible to determine that the storage function $V_{1b}(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}})$ is positive definite and radially unbounded. By employing Property 1 the time derivative of the storage function (13) along the trajectories of the closed loop (14) results in

$$\dot{V}_{1b}(\dot{\boldsymbol{q}}(t),\tilde{\boldsymbol{q}}(t)) = (\dot{\boldsymbol{q}} - \alpha\,\mathrm{sat}(\tilde{\boldsymbol{q}}))^T\boldsymbol{\tau}'' - \varphi(\dot{\boldsymbol{q}}(t),\tilde{\boldsymbol{q}}(t))$$

This, shows the state strict passivity of $\boldsymbol{\tau}''$ to the output $(\dot{\boldsymbol{q}} - \alpha\,\mathrm{sat}(\tilde{\boldsymbol{q}}))$, provided that $\varphi(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}})$ be positive definite.

Right a way we will prove that the function $\varphi(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}})$ is positive definite. By using the next inequalities:

$$\begin{array}{rcl}
-\dot{\boldsymbol{q}}^T K_v \mathbf{SAT}(\dot{\boldsymbol{q}}) & \leq & -\lambda_{\rm m}\{K_v\}\,\|\mathbf{SAT}(\dot{\boldsymbol{q}})\|^2 \\
-\dot{\boldsymbol{q}}^T F_v \dot{\boldsymbol{q}} & \leq & -\lambda_{\rm m}\{F_v\}\,\|\dot{\boldsymbol{q}}\|^2 \\
-\alpha\dot{\tilde{\mathbf{sat}(\boldsymbol{q})}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} & \leq & \alpha\lambda_{\rm M}\{M\}\,\|\dot{\boldsymbol{q}}\|^2 \\
-\alpha\mathbf{sat}(\tilde{\boldsymbol{q}})^T C(\boldsymbol{q},\dot{\boldsymbol{q}})^T\dot{\boldsymbol{q}} & \leq & \alpha k_{c_1}\sqrt{n}\,\|\dot{\boldsymbol{q}}\|^2 \\
\alpha\mathbf{sat}(\tilde{\boldsymbol{q}})^T K_v\mathbf{SAT}(\dot{\boldsymbol{q}}) & \leq & \alpha\lambda_{\rm M}\{K_v\}\,\|\mathbf{sat}(\tilde{\boldsymbol{q}})\| \\
 & & \times\,\|\mathbf{SAT}(\dot{\boldsymbol{q}})\| \qquad (15) \\
-\alpha\mathbf{sat}(\tilde{\boldsymbol{q}})^T[\boldsymbol{g}(\boldsymbol{q}_d) - \boldsymbol{g}(\boldsymbol{q})] & \leq & \alpha k_{h_2}\,\|\mathbf{sat}(\tilde{\boldsymbol{q}})\|^2 \\
-\mathbf{sat}(\tilde{\boldsymbol{q}})^T K_p\mathbf{SAT}(\tilde{\boldsymbol{q}}) & \leq & -\alpha\lambda_{\rm m}\{K_p\}\,\|\mathbf{sat}(\tilde{\boldsymbol{q}})\|^2
\end{array}$$

with k_{h_2} given in [23] as

$$k_{h_2} = \frac{2k'}{\tanh(\frac{2k'}{k_g})}$$

—for the case when $\mathrm{sat}(\cdot) = \tanh(\cdot)$—, we obtain the next expression

$$\begin{array}{rcl}
-\varphi(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}}) & \leq & -\left[\,\lambda_{\rm m}\{F_v\} - \alpha(\lambda_{\rm M}\{M\} + \sqrt{n}k_{c_1})\right]\ \|\dot{\boldsymbol{q}}\|^2 \\
 & & -\alpha\begin{bmatrix}\|\mathbf{sat}(\tilde{\boldsymbol{q}})\| \\ \|\mathbf{SAT}(\dot{\boldsymbol{q}})\|\end{bmatrix}^T Q \begin{bmatrix}\|\mathbf{sat}(\tilde{\boldsymbol{q}})\| \\ \|\mathbf{SAT}(\dot{\boldsymbol{q}})\|\end{bmatrix}
\end{array}$$

where

$$Q = \begin{bmatrix} \lambda_{\rm m}\{K_p\} - k_{h_2} & \frac{\lambda_{\rm M}\{K_v\}}{2} \\ \frac{\lambda_{\rm M}\{K_v\}}{2} & \frac{\lambda_{\rm m}\{K_v\}}{\alpha} \end{bmatrix},$$

which will be positive definite if:

$$\frac{\lambda_{\rm m}\{F_v\}}{\lambda_{\rm M}\{M\} + \sqrt{n}k_{c_1}} > \alpha. \tag{16}$$

$$\lambda_{\rm m}\{K_p\} > k_{h_2} \tag{17}$$

$$\frac{4[\lambda_{\rm m}\{K_p\} - k_{h_2}]\lambda_{\rm m}\{K_v\}}{\lambda^2_{\rm M}\{K_v\}} > \alpha. \tag{18}$$

Then $\varphi(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}}) > 0$ will be positive definite, provided that (16), (17) and (18), be satisfied. This completes the proof. ♢

Proposition 2. The system (see H_2 in Figure 2)

$$\begin{array}{rcl}
\dot{\boldsymbol{w}} & = & \alpha\,\mathbf{sat}(\tilde{\boldsymbol{q}}) - \dot{\boldsymbol{q}} \\
\boldsymbol{z} & = & [-K_i\mathbf{SAT}(\boldsymbol{w}) + \boldsymbol{g}(\boldsymbol{q}_d)]
\end{array}$$

is passive from the input $\dot{\boldsymbol{q}} - \alpha\,\mathbf{sat}(\tilde{\boldsymbol{q}})$ to the output $\boldsymbol{z}$ with a radially unbounded non negative storage function given by

$$V_2(\boldsymbol{w} - {K_i}^{-1}\boldsymbol{g}(\boldsymbol{q}_d)) = \sum_{i=1}^{n} k_{ii}\int_{{k_{ii}}^{-1}g_i(\boldsymbol{q}_d)}^{w_i}[\mathrm{SAT}(\xi_i) - {k_{ii}}^{-1}g_i(\boldsymbol{q}_d)]\,d\xi_i, \tag{19}$$

which has an unique and global minimum at $\boldsymbol{w} = K_i^{-1}\boldsymbol{g}(\boldsymbol{q}_d)$ —provided that $\lambda_{\min}\{K_i\} > \max_i \bar{g}_i$—. That means:

$$\int_0^T(\dot{\boldsymbol{q}} - \alpha\,\mathbf{sat}(\tilde{\boldsymbol{q}}))^T\boldsymbol{z}\,dt \geq -V_2(0) \tag{20}$$

Proof. By using the Property 4 it is possible to determine that the storage function V_2 is non negative with a minimum point in $\boldsymbol{w}^* = {K_i}^{-1}\boldsymbol{g}(\boldsymbol{q}_d)$ if $\lambda_{\min}\{K_i\} > \max_i \bar{g}_i$. The time derivative of the storage function (19) results in

$$\begin{array}{rcl}
\dot{V}_2 & = & [-K_i\mathbf{SAT}(\boldsymbol{w}) + \boldsymbol{g}(\boldsymbol{q}_d)]^T[\dot{\boldsymbol{q}} - \alpha\,\mathbf{sat}(\tilde{\boldsymbol{q}})] \\
 & = & \boldsymbol{z}^T(-\dot{\boldsymbol{w}}).
\end{array} \tag{21}$$

By integrating from 0 a T, in a direct form we have (20). This shows the passivity from $-\dot{\boldsymbol{w}} = (\dot{\boldsymbol{q}} - \alpha\,\mathbf{sat}(\tilde{\boldsymbol{q}}))$ to the output $\boldsymbol{z}$.

Right a way, we present a theorem that allows to conclude global asymptotic stability for the equilibrium of an unforced feedback system, which is composed by the feedback interconnection of a state strictly passive system with a passive system. ♢

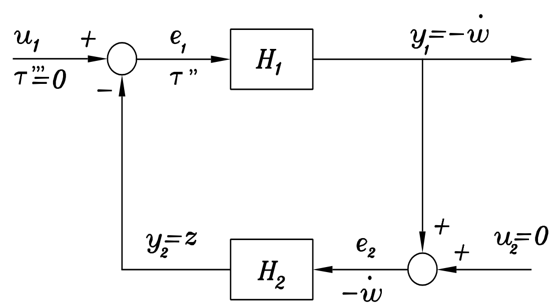

Fig. 3. Feedback connection

Theorem 1. Consider the feedback system of Figure 3 where H_1 and H_2 are dynamical systems of the form

$$\begin{array}{rcl}
\dot{\boldsymbol{x}}_i & = & \boldsymbol{f}_i(\boldsymbol{x}_i,\boldsymbol{e}_i) \\
\boldsymbol{y}_i & = & \boldsymbol{h}_i(\boldsymbol{x}_i,\boldsymbol{e}_i)
\end{array}$$

for $i = 1, 2$, where $\boldsymbol{f}_i : \mathbb{R}^{m_i}\times\mathbb{R}^n \to \mathbb{R}^{m_i}$ and $\boldsymbol{h}_i : \mathbb{R}^{m_i}\times\mathbb{R}^n \to \mathbb{R}^n$ are supposed sufficiently smooth such that the system is well–defined. Also we assume $\boldsymbol{f}_1(\mathbf{0},\boldsymbol{e}_1) = \mathbf{0} \Rightarrow \boldsymbol{e}_1 = \mathbf{0}$, $\boldsymbol{f}_2({\boldsymbol{x}_2}^*,\mathbf{0}) = \mathbf{0}$, $\boldsymbol{h}_1(\mathbf{0},\mathbf{0}) = \mathbf{0}$. and $\boldsymbol{h}_2({\boldsymbol{x}_2}^*,\mathbf{0}) = \mathbf{0}$. The system has the same number of inputs and outputs. Suppose the feedback system has a well–defined state–space model

$$\begin{array}{rcl}
\dot{\boldsymbol{x}} & = & \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u}) \\
\boldsymbol{y} & = & \boldsymbol{h}(\boldsymbol{x},\boldsymbol{u})
\end{array} \tag{22}$$

where

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix}, \; \boldsymbol{u} = \begin{bmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \end{bmatrix}, \boldsymbol{y} = \begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{bmatrix}$$

$\boldsymbol{f}$ and $\boldsymbol{h}$ are sufficiently smooth, $\boldsymbol{f}(\boldsymbol{x}^*, \boldsymbol{0}) = \boldsymbol{0}$, and $\boldsymbol{h}(\boldsymbol{x}^*, \boldsymbol{0}) = \boldsymbol{0}$. Let H_1 be a state strictly passive system with a positive definite storage function $V_1(\boldsymbol{x}_1)$ and state dissipation rate $\rho_1\psi_1(\boldsymbol{x}_1)$ and H_2 be a passive and equilibrium–state observable system with a non negative storage function $V_2(\boldsymbol{x}_2 - \boldsymbol{x}_2^*)$ with a unique minimum in $\boldsymbol{x}_2{}^*$; that is,

$$\begin{aligned} \boldsymbol{e}_1^T \boldsymbol{y}_1 &\geq \dot{V}_1(\boldsymbol{x}_1) + \rho_1\psi_1(\boldsymbol{x}_1) \\ \boldsymbol{e}_2^T \boldsymbol{y}_2 &\geq \dot{V}_2(\boldsymbol{x}_2 - \boldsymbol{x}_2^*) \end{aligned}$$

Then the equilibrium $\boldsymbol{x}^*$ of

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{0}) \tag{23}$$

is asymptotically stable. If $V_1(\boldsymbol{x}_1)$ and $V_2(\boldsymbol{x}_2 - \boldsymbol{x}_2^*)$ are radially unbounded then the equilibrium of (23) will be globally asymptotically stable.

Proof. Take $\boldsymbol{u}_1 = \boldsymbol{u}_2 = \boldsymbol{0}$. In this case $\boldsymbol{e}_1 = -\boldsymbol{y}_2$ and $\boldsymbol{e}_2 = \boldsymbol{y}_1$. Using $V(\boldsymbol{x} - \boldsymbol{x}^*) = V_1(\boldsymbol{x}_1) + V_2(\boldsymbol{x}_2 - \boldsymbol{x}_2^*)$ as a Lyapunov function candidate for the closed–loop system, we have

$$\begin{aligned} \dot{V}(\boldsymbol{x} - \boldsymbol{x}^*) &= \dot{V}_1(\boldsymbol{x}_1) + \dot{V}_2(\boldsymbol{x}_2 - \boldsymbol{x}_2^*) \\ &\leq \boldsymbol{e}_1^T \boldsymbol{y}_1 - \rho_1\psi_1(\boldsymbol{x}_1) + \boldsymbol{e}_2^T \boldsymbol{y}_2 \\ &= -\rho_1\psi_1(\boldsymbol{x}_1) \leq 0, \end{aligned}$$

which shows that the equilibrium $\boldsymbol{x}^*$ of the closed-loop system is stable. To prove asymptotic stability we use the LaSalle's invariance principle and the equilibrium–state observability of the system H_2. It remains to demonstrate that $\boldsymbol{x} = \boldsymbol{x}^*$ is the largest invariant set in $\Omega = \{\boldsymbol{x} \in \mathbb{R}^{m_1+m_2} : \dot{V}(\boldsymbol{x} - \boldsymbol{x}^*) = 0\}$. To this end, in the search of the largest invariant set, we have to show that $\dot{V}(\boldsymbol{x} - \boldsymbol{x}^*) \equiv 0 \Rightarrow \boldsymbol{x}^* \equiv \boldsymbol{0} \in \mathbb{R}^{2n}$. From this we have that $\dot{V}(\boldsymbol{x} - \boldsymbol{x}^*) = 0 \Rightarrow 0 \leq -\rho_1\psi_1(\boldsymbol{x}_1) \leq 0 \Rightarrow -\rho_1\psi_1(\boldsymbol{x}_1) = 0$. Besides

$$\rho_1 > 0 \Rightarrow \psi_1(\boldsymbol{x}_1) \equiv 0 \Rightarrow \boldsymbol{x}_1 \equiv 0.$$

Now, as $\boldsymbol{x}_1 \equiv 0 \Rightarrow \dot{\boldsymbol{x}}_1 = \boldsymbol{f}_1(\boldsymbol{x}_1, \boldsymbol{e}_1) \equiv 0$ and in agreement with the assumption about $\boldsymbol{f}_1(\cdot, \cdot)$ in the sense that $\boldsymbol{f}_1(\boldsymbol{0}, \boldsymbol{e}_1) = \boldsymbol{0} \Rightarrow \boldsymbol{e}_1 = \boldsymbol{0}$, we have $\boldsymbol{e}_1 \equiv \boldsymbol{0} \Rightarrow \boldsymbol{y}_2 \equiv \boldsymbol{0}$. Also $\boldsymbol{x}_1 \equiv \boldsymbol{0}$, $\boldsymbol{e}_1 \equiv \boldsymbol{0} \Rightarrow \boldsymbol{y}_1 \equiv \boldsymbol{0}$ (owing to assumption $\boldsymbol{h}_1(\boldsymbol{0}, \boldsymbol{0}) = 0$). Finally, $\boldsymbol{y}_1 \equiv \boldsymbol{0} \Rightarrow \boldsymbol{e}_2 \equiv \boldsymbol{0}$, and

$$\boldsymbol{e}_2 \equiv \boldsymbol{0} \text{ and } \boldsymbol{y}_2 \equiv \boldsymbol{0} \Rightarrow \boldsymbol{x}_2 \equiv \boldsymbol{x}^*$$

in agreement with the equilibrium–state observability of H_2. This shows that the largest invariant set in Ω is the equilibrium, $\boldsymbol{x}^* = \begin{bmatrix} \boldsymbol{x}_1{}^T & \boldsymbol{x}_2{}^{*T} \end{bmatrix}$ hence, by using the Krasovskii–LaSalle's theorem, we conclude asymptotic stability of the equilibrium of the unforced closed-loop system (23). If $V(\boldsymbol{x})$ is radially unbounded then the equilibrium will be globally asymptotically stable.

$\diamondsuit$

In relation to Theorem 1 and considering that $\boldsymbol{x}_1 = \begin{bmatrix} \tilde{\boldsymbol{q}}^T & \dot{\boldsymbol{q}}^T \end{bmatrix}^T$, $\boldsymbol{x}_2 = \boldsymbol{w}^T$, $\boldsymbol{e}_2 = \boldsymbol{y}_1 = -\dot{\boldsymbol{w}}$, $\boldsymbol{y}_2 = \boldsymbol{z}$, $\boldsymbol{e}_1 = \boldsymbol{\tau}''$, $V_1(\boldsymbol{x}) = V_{1b}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}})$, $V_2(\boldsymbol{x}_2 - \boldsymbol{x}_2^*) = V_2(\boldsymbol{w} - K_i{}^{-1}\boldsymbol{g}(\boldsymbol{q}_d))$, $\boldsymbol{u}_1 = \boldsymbol{\tau}''' = 0$, $\boldsymbol{u}_2 = 0$. The closed-loop system equation (10) leads to (22) (see Figure 2).

By using the Propositions 1, 2 and Theorem 1 we can prove the following:

Proposition 3. Consider the saturated nonlinear PID regulator (9) in closed-loop with the robot dynamics (1). The closed-loop system can be represented by an interconnected system (see Figure 2), which satisfies the following conditions

- A1. The system in the forward path defines a state strictly passive mapping with a radially unbounded positive definite storage function given by (13), provided that $\lambda_{\min}\{K_p\} > k_{h_2}$.
- A2. The system in the feedback path defines an equilibrium state observable passive mapping with a non negative and radially unbounded storage function given by (19), provided that $\lambda_{\min}\{K_i\} > \max_i \bar{g}_i$.

Besides, the equilibrium $\begin{bmatrix} \tilde{\boldsymbol{q}}^T & \dot{\boldsymbol{q}}^T & \boldsymbol{w}^T \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & K_i^{-1}\boldsymbol{g}(\boldsymbol{q}_d) \end{bmatrix}^T \in \mathbb{R}^{3n}$ of the closed-loop system (10) is globally asymptotically stable.

Furthermore the applied torques are bounded by $|\tau_i| \leq \tau_i^{\max}$ for $i = 1, 2, 3...n$.

$\diamondsuit$

V. SIMULATION RESULTS

Using the SIMNON, we tested our algorithm in the two revolute jointed robot manipulator used in [24]. The desired joint positions were chosen as $q_{d1} = 90°$ and $q_{d2} = 60°$. The gain was tuned as $K_p = \mathrm{diag}\{40, 39\}$ [Nm/rad], $K_i = \mathrm{diag}\{100, 100\}$ [Nm/rad sec] and $K_v = \mathrm{diag}\{12, 12\}$ [Nm sec/rad]. The maximum torques supplied by the actuators are $\tau_1^{\max} = 15$ [Nm] and $\tau_2^{\max} = 4$ [Nm]. The parameters to be used are: $\lambda_{\mathrm{M}}\{M(\boldsymbol{q})\} = 0.361$ [kg m^2], $\lambda_{\mathrm{m}}\{M(\boldsymbol{q})\} = 0.011$ [kg m^2], $k_g = 23.94$ [kg m^2/sec^2], $k_{c_1} = 0.049$ [kg m^2], $k_{h_2} = 31.43$ [Nm], $\lambda_{\mathrm{m}}\{F_v\} = 0.1713$ and $\alpha = 0.397$ [sec^{-1}].

The Figure 4 shows how the position errors converge to

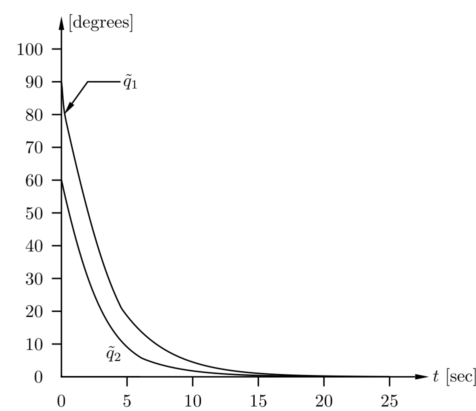

Fig. 4. Joint position errors for the saturated PID Control

zero and the Figure 5 shows the torques for a period of six

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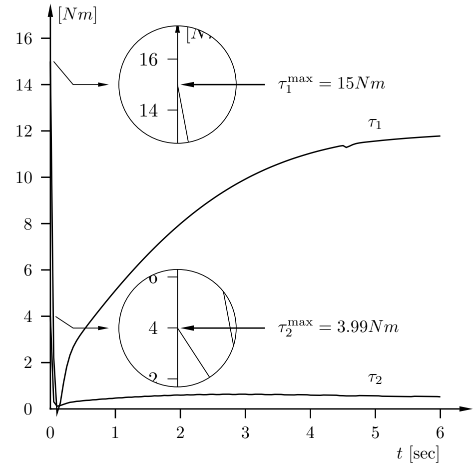

Fig. 5. Applied torque using the saturated PID Control

second. The proposed saturated nonlinear PID controller yields control inputs $|\tau_1| < \tau_1^{\max} = 15$ [Nm] and $|\tau_2| < \tau_2^{\max} = 4$ [Nm].

$\diamondsuit$

VI. CONCLUSIONS

In this paper we have proposed a saturated nonlinear PID regulator to solve the global regulation problem of robot manipulators with bounded torques.

By using a passivity based approach, we have presented a global asymptotic stability analysis of the closed loop system.

It has been proved that the passive structure of the rigid robot is preserved in closed loop with the saturated feedback of position and velocity from a new input torque $\boldsymbol{\tau}''$ to the output $-\dot{\boldsymbol{w}} = (\dot{\boldsymbol{q}} - \alpha\,\mathbf{sat}(\tilde{\boldsymbol{q}}))$. Besides, the feedback corresponding to the saturated integral action defines a passive mapping.

Based on the above reasoning we show that the proposed saturated nonlinear PID regulator in closed-loop with the robot manipulator, can be represented as a feedback system composed by two blocks, in which, the feedforward system is state strictly passive and the feedback system is passive and equilibrium–state observable.

Global asymptotic stability of the equilibrium of the closed-loop system is given in a direct way using a passivity theorem, which is an adaptation of a passivity theorem presented in the literature of passive systems.

It is also guaranteed that, regardless of initial conditions, the delivered torques evolve inside prescribed limits.

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